Galaxy Number Counts

March 30, 2024

Linwei Qiu

Sun Yet-sen University

qiulw3@mail2.sysu.edu.com

- FLRW Metric
- Perturbed Photon Paths
  - Temporal component: Sachs-Wolfe effect
  - Spatial components: Gravitational lensing effect
- Galaxy Number Counts Without Bias
  - Perturbation in Redshift Space
  - Volume Perturbation
- Observed Luminosity Distance
  - Flux Limit
  - Perturbed Luminosity Distance
- Bias
  - Clustering Bias
  - Evolution Bias
  - Magnification Bias
- Caculation

FLRW Metric

Unperturbed Universe

The FLRW Metric

$$ds^2 = a^2(\eta)[-d\eta^2 + \gamma_{ij}dx^i dx^j] \tag{1}$$

$$\gamma_{ij}dx^i dx^j = dr^2 + \chi^2(r)(d\theta^2 + \sin^2\theta d\varphi^2) \tag{2}$$

$$\chi(r) = \begin{cases} r & K = 0 \\ \frac{1}{\sqrt{K}}\sin(\sqrt{K}r) & k > 0 \\ \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}r) & k < 0 \end{cases} \tag{3}$$

In flat universe we have

$$ds^2 = a^2(\eta)[-d\eta^2 + \delta_{ij}dx^i dx^j] \tag{4}$$

$$r = a(\eta_0 - \eta) = \int_0^z \frac{dz}{H} \tag{5}$$

FLRW Metric

Perturbed Universe

Newtonian Gauge

$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)\gamma_{ij}dx^i dx^j] \tag{6}$$

$$\gamma_{ij}dx^i dx^j = dr^2 + \chi^2(r)(d\theta^2 + \sin^2\theta d\varphi^2) \tag{7}$$

$$\chi(r) = \begin{cases} r & K = 0 \\ \frac{1}{\sqrt{K}}\sin(\sqrt{K}r) & k > 0 \\ \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}r) & k < 0 \end{cases} \tag{8}$$

In flat universe we have

$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j] \tag{9}$$

$$\sqrt{-g} = a^4(1+\Psi-3\Phi) \tag{10}$$

$$u^\mu = n^\mu = \frac{1}{a}(1+\Psi, u^i) \tag{11}$$

Perturbed Photon Paths

Temporal component: Sachs-Wolfe effect

To condisder the redshift we can work in comoving coordinates

$$d\tilde{s}^2 = a^2 ds^2 \tag{12}$$

The observer receives the photon redshifted by a factor

$$z + 1 = \frac{(\tilde{u} \cdot \tilde{n})_s}{(\tilde{u} \cdot \tilde{n})_o} = \frac{a(\eta_o)}{a(\eta_s)} \frac{(u \cdot n)_s}{(u \cdot n)_o} \tag{13}$$

where n^μ is the 4-velocity of photon and u^μ is the peculiar velocity. The first order geodesic equation in flat universe yeilds

$$\frac{d\delta n^i}{d\lambda} - (\Psi + \Phi)' + 2\frac{d\Psi}{d\lambda} = 0 \tag{14}$$

$$\frac{d\delta n^i}{d\lambda} - \partial^i(\Psi + \Phi) - 2\frac{d(n^i\Phi)}{d\lambda} = 0 \tag{15}$$

Solution:

$$\delta n^0|_s^o = -2\Psi|_s^o + \int_s^o (\Psi + \Phi)' d\lambda \tag{16}$$

$$\delta n^i|_s^o = 2n^i\Phi|_s^o + \int_s^o \partial^i(\Psi + \Phi) d\lambda \tag{17}$$

Perturbed Photon Paths

Temporal component: Sachs-Wolfe effect

With $n^\mu = (n^0 + \delta n^0, n^i + \delta n^i)$, $u^\mu = (1 - \Psi, v^i)$Redshift

$$
\begin{aligned}
z + 1 &= \frac{a(\eta_o)}{a(\eta_s)} \frac{(\mathbf{v} \cdot \mathbf{n} + \Psi - \delta n^0 - 1)_s}{(\mathbf{v} \cdot \mathbf{n} + \Psi - \delta n^0 - 1)_o} \\
&= \frac{a(\eta_o)}{a(\eta_s)} \left[1 + \Psi|_s^o + \mathbf{v} \cdot \mathbf{n}|_s^o - \int_s^o (\Psi + \Phi)' d\lambda \right] \\
&= \frac{a(\eta_o)}{a(\eta_s)} \left[1 + \Psi|_s^o + \mathbf{v} \cdot \mathbf{n}|_s^o + \int_0^{r_s} (\Psi + \Phi)' dr \right]
\end{aligned}
\tag{18}
$$

So

$$
\frac{\delta z}{1 + z} = [\Psi + \mathbf{v} \cdot \mathbf{n}]_s^o + \int_0^{r_s} (\Psi + \Phi)' dr \tag{19}
$$

- RSD: $[\Psi + \mathbf{v} \cdot \mathbf{n}]_s^o$
- SW effect: $\int_0^{r_s} (\Psi + \Phi)' dr$

Perturbed Photon Paths[1, 2]

Spatial components: Gravitational lensing effect

The path of null geodesics

$$
\begin{aligned}
ds^2 &= a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)\gamma_{ij}dx^i dx^j] = 0 \\
\Rightarrow &(1+2\Psi+2\Phi)d\eta^2 + \gamma_{ij}dx^i dx^j = 0
\end{aligned}
$$

Definition (The Wely potential and the conformally-related metric)

$$\Psi_W = \frac{1}{2}(\Psi + \Phi) \tag{20}$$

$$d\tilde{s}^2 = (1+4\Psi_W)d\eta^2 + \gamma_{ij}dx^i dx^j = 0 \tag{21}$$

We have ($\bar{\Gamma}^k_{ij}$ are the Christoffel symbols of the unperturbed metric γ_{ij})

$$
\begin{aligned}
&\tilde{\Gamma}^0_{00} = 2\partial_\eta \Psi_W && \tilde{\Gamma}^0_{0i} = 2\partial_i \Psi_W && \tilde{\Gamma}^0_{ij} = 0 \\
&\tilde{\Gamma}^i_{00} = 2\gamma^{ij}\partial_j \Psi_W && \tilde{\Gamma}^i_{j0} = 0 && \tilde{\Gamma}^k_{ij} = \bar{\Gamma}^k_{ij}
\end{aligned}
$$

Perturbed Photon Paths

Spatial components: Gravitational lensing effect

The geodesics equations

$$\frac{d^2\eta}{d\lambda^2} + 2\left(\frac{d\eta}{d\lambda}\right)^2 \frac{d\Psi_W}{d\eta} + 2\frac{d\eta}{d\lambda}\frac{dx^i}{d\lambda}\frac{\partial\Psi_W}{\partial x^i} = 0 \tag{22}$$

$$\frac{d^2x^i}{d\lambda^2} + 2\gamma^{ij}\frac{\partial\Psi_W}{\partial x^j} + \bar{\Gamma}^i_{jk}\frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda} = 0 \tag{23}$$

We can eliminate the affine parameter λ in favour of η

$$\frac{d^2x^i}{d\eta^2} - 2\frac{dx^i}{d\eta}\left(\frac{d\Psi_W}{d\eta} + \frac{dx^j}{d\eta}\frac{\partial\Psi_W}{\partial x^j}\right) + 2\gamma^{ij}\frac{\partial\Psi_W}{\partial x^j} + \bar{\Gamma}^i_{jk}\frac{dx^j}{d\eta}\frac{dx^k}{d\eta} = 0 \tag{24}$$

We have

$$\frac{d^2r}{d\eta^2} + 2\frac{d\Psi_W}{d\eta} = 0 \tag{25}$$

$$\frac{d^2\theta}{d\eta^2} - 2\frac{d\ln\chi(r)}{dr}\frac{d\theta}{d\eta} + \frac{2}{\chi^2(r)}\frac{\partial\Psi_W}{\partial\theta} = 0 \tag{26}$$

$$\frac{d^2\varphi}{d\eta^2} - 2\frac{d\ln\chi(r)}{dr}\frac{d\varphi}{d\eta} + \frac{2}{\chi^2(r)\sin^2\theta}\frac{\partial\Psi_W}{\partial\varphi} = 0 \tag{27}$$

Perturbed Photon Paths

Spatial components: Gravitational lensing effect

Initial Condition:

- Observer position: $\mathbf{r} = 0$, time receive the photon $\eta = \eta_0$
- Unperturbed photon trajectory: $\theta = \theta_0$, $\varphi = \varphi_0$
- Neglect the perturbation at observer

Solutions:

$$r(\eta) = \eta_0 - \eta + 2\int_{\eta}^{\eta_0} \Psi_W d\eta' \tag{28}$$

$$\theta(\eta) = \theta_0 - 2\int_{\eta}^{\eta_0} \frac{\chi(\eta' - \eta)\partial_\theta \Psi_W(\eta', \eta_0 - \eta', \theta_0, \varphi_0)}{\chi(\eta_0 - \eta)\chi(\eta_0 - \eta')} d\eta' \tag{29}$$

$$\varphi(\eta) = \varphi_0 - \frac{2}{\sin^2\theta_0}\int_{\eta}^{\eta_0} \frac{\chi(\eta' - \eta)\partial_\varphi \Psi_W(\eta', \eta_0 - \eta', \theta_0, \varphi_0)}{\chi(\eta_0 - \eta)\chi(\eta_0 - \eta')} d\eta' \tag{30}$$

We found that $\frac{d\eta}{d\lambda} = -\frac{dr}{d\lambda} = 1 + O(\Psi)$

Galaxy Number Counts[1, 3]

What can we observe

Redshifts z, Directions $\mathbf{n}$, Number N
Number Count in Redshift Space: $N(\mathbf{n}, z)$
Number Count Flucuation

$$\Delta_N(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)}$$

What we need for models

Matter Denisty: $\rho(\mathbf{x}, \eta)$
Matter Denisty Flucuation

$$\delta_z(\mathbf{x}, \eta) = \frac{\delta\rho}{\rho}\bigg|_{\mathbf{x},\eta} = \frac{\rho(\mathbf{x}, \eta) - \bar{\rho}(\eta)}{\bar{\rho}(\eta)}$$

Redshift Matter Denisty Flucuation

$$\delta_z(\mathbf{n}, z) = \frac{\delta\rho}{\rho}\bigg|_{\mathbf{n},z} = \frac{\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)}$$

Galaxy Number Counts

First we show the number count fluctuation given by Durrer

$$\begin{aligned}\Delta(\mathbf{n},z) &= \delta_m + \Phi + \Psi + \frac{1}{\mathcal{H}}[\Phi' + \partial_r(\mathbf{V}\cdot\mathbf{n})] \\ &+ \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2}{r_s\mathcal{H}}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_s}(\Psi' + \Phi')dr\right) \\ &+ \frac{1}{r_s}\int_0^{r_s}\left(2 - \frac{r_s - r}{r}\nabla_\Omega^2\right)(\Psi + \Phi)dr\end{aligned} \tag{31}$$

where ∇_Ω^2 denotes the angular part of the Laplacian

$$\nabla_\Omega^2 = \cot\theta\partial_\theta + \partial_\theta^2 + \frac{1}{\sin^2\theta}\partial_\varphi^2 \tag{32}$$

Lensing focusing

$$\kappa = \int_0^{r_s}\frac{r_s - r}{2r_s r}\nabla_\Omega^2(\Psi + \Phi)dr \tag{33}$$

Galaxy Number Counts

Perturbation in Redshift Space

Number count flucuation in redshift space[1]

$$\Delta_N = \delta_z + \frac{\delta V}{V} \tag{34}$$

Relation between $\delta_z(\mathbf{n}, z)$ and $\delta_m(\mathbf{x}, t)$

$$\delta_z = \frac{\delta\rho}{\bar{\rho}} - \frac{d\bar{\rho}}{dz}\frac{\delta z}{\bar{\rho}} = \delta_m - 3\frac{\delta z}{1+z} \tag{35}$$

Bonus:

$$\frac{d\bar{\rho}}{dz} = 3\frac{\bar{\rho}}{1+z} \tag{36}$$

Proof: For $1 + z = a_0/a$, $\rho_m \propto a^{-3}$, we have

$$\frac{d\bar{\rho}}{dz} = \frac{d\bar{\rho}}{da}\frac{da}{dz} = 3\bar{\rho}a = 3\frac{\bar{\rho}}{1+z}$$

[1] Before introducing bias, we assume that matter density fluctuation is equal to galaxy number density fluctuation, $\delta_m = \delta_s$

The spatial volume element has to be defined an observer moving with 4-velocity u_o^μ as

$$\begin{aligned} dV &= \sqrt{-g}\varepsilon_{\mu\nu\alpha\beta}u^\mu dx^\nu dx^\alpha dx^\beta \\ &= \sqrt{-g}\varepsilon_{\mu\nu\alpha\beta}u^\mu \frac{\partial x^\nu}{\partial z}\frac{\partial x^\alpha}{\partial \theta_s}\frac{\partial x^\beta}{\partial \varphi_s} dz d\theta_s d\varphi_s \\ &= \sqrt{-g}\varepsilon_{\mu\nu\alpha\beta}u^\mu \frac{\partial x^\nu}{\partial z}\frac{\partial x^\alpha}{\partial \theta_s}\frac{\partial x^\beta}{\partial \varphi_s}\left|\frac{\partial(\theta_s,\varphi_s)}{\partial(\theta_o,\varphi_o)}\right| dz d\theta_o d\varphi_o \\ &= v(z,\theta_o,\varphi_o) dx^\nu d\Omega_o \end{aligned} \tag{37}$$

The volume perturbation can be defined as

$$\frac{\delta V}{\bar{V}} = \frac{\delta v}{\bar{v}} \tag{38}$$

Galaxy Number Counts

Volume Perturbation

The perturbed angles at the source: $\theta_s = \theta_o + \delta\theta$ and $\varphi_s = \varphi_o + \delta\varphi$

$$\left|\frac{\partial(\theta_s, \varphi_s)}{\partial(\theta_o, \varphi_o)}\right| = 1 + \frac{\partial\delta\theta}{\partial\partial\theta} + \frac{\partial\delta\varphi}{\partial\partial\varphi} \tag{39}$$

We have

$$v = a^3(1+\Psi-3\Phi)\left[\frac{dr}{dz}r^2\frac{\sin\theta_s}{\sin\theta_o}\left(1 + \frac{\partial\delta\theta}{\partial\partial\theta} + \frac{\partial\delta\varphi}{\partial\partial\varphi}\right)(1 - \Psi + V_r)\right] \tag{40}$$

where ($\frac{d\bar{r}}{d\bar{z}} = \frac{a}{\mathcal{H}}$)

$$\frac{dr}{dz} = \frac{d\bar{r}}{d\bar{z}} + \frac{d\delta r}{d\bar{z}} - \frac{d\delta z}{d\bar{z}}\frac{d\bar{r}}{d\bar{z}} = \frac{a}{\mathcal{H}}\left(1 - \frac{d\delta z}{d\bar{z}} + \frac{d\delta r}{d\bar{r}}\right)$$

$$\frac{\sin\theta_s}{\sin\theta_o} = \frac{\sin(\theta_o + \delta\theta)}{\sin\theta_o} = 1 + \cot\theta_o\delta\theta$$

Galaxy Number Counts

Volume Perturbation

We have

$$\bar{v}(\bar{z}) = \frac{\bar{r}^2}{(1+\bar{z})^4\mathcal{H}} \tag{41}$$

$$\bar{v}(z) = \bar{v}(\bar{z}) + \frac{d\bar{v}}{d\bar{z}}\delta z \tag{42}$$

$$\frac{d\bar{v}}{d\bar{z}} = \frac{\bar{v}}{1+\bar{z}}\left(\frac{2}{\bar{r}\mathcal{H}} - 4 + \frac{\mathcal{H}'}{\mathcal{H}^2}\right) \tag{43}$$

Density fluctuation

$$\begin{aligned}\frac{\delta v}{\bar{v}} &= \frac{v(z)-\bar{v}(\bar{z})}{\bar{v}(\bar{z})} \\ &= -3\Phi + \left(\cot\theta_o + \frac{\partial}{\partial\theta}\right)\delta\theta + \frac{\partial\delta\varphi}{\partial\varphi} - \mathbf{v}\cdot\mathbf{n} + \frac{2\delta r}{r} - \frac{d\delta r}{d\eta} \\ &+ \frac{1}{\mathcal{H}(1+\bar{z})}\frac{d\delta z}{d\eta} - \left(\frac{\delta z}{1+\bar{z}}\right)\left(\frac{2}{\bar{r}\mathcal{H}} - 4 + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\end{aligned} \tag{44}$$

> ***Goal***
>
> Find the expression of: δr, $\delta\theta$, $\delta\varphi$

Galaxy Number Counts

Volume Perturbation

δr, $\delta\theta$, $\delta\varphi$ have been caculated in (28), (29), (30). In flat universe we get

$$\delta r(r_s) = \int_0^{r_s} (\Psi + \Phi) dr \tag{45}$$

$$\delta\theta(r_s) = -\int_0^{r_s} \frac{r_s - r}{r_s r} \partial_\theta (\Psi + \Phi) dr \tag{46}$$

$$\delta\varphi(r_s) = -\frac{1}{\sin^2\theta_0} \int_0^{r_s} \frac{r_s - r}{r_s r} \partial_\varphi (\Psi + \Phi) dr \tag{47}$$

Density fluctuation

$$\begin{aligned} \frac{\delta v}{\bar{v}} &= -2(\Psi + \Phi) - 4\mathbf{v}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\left(n^i \partial_i \Psi + \Phi' + \frac{d(\mathbf{v}\cdot\mathbf{n})}{d\eta}\right) \\ &+ \left(\frac{2}{\bar{r}\mathcal{H}} + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left(\mathbf{v}\cdot\mathbf{n} + \Psi + \int_0^{r_s} (\Psi + \Phi)' dr\right) \\ &+ \frac{2}{r_s}\int_0^{r_s} (\Psi + \Phi) dr - 3\int_0^{r_s} (\Psi + \Phi)' dr - \int_0^{r_s} \frac{r_s - r}{r_s r} \nabla_\Omega^2 (\Psi + \Phi) dr \end{aligned} \tag{48}$$

Galaxy Number Counts

Galaxy Number Density Flucuation

$$\begin{aligned}\Delta(\mathbf{n}, z) &= \delta_m + \Psi - 2\Phi + \frac{1}{\mathcal{H}}[\Phi' + \partial_r(\mathbf{V}\cdot\mathbf{n})] \\ &+ \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2}{r_s\mathcal{H}}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_s}(\Psi' + \Phi')dr\right) \\ &+ \frac{1}{r_s}\int_0^{r_s}\left(2 - \frac{r_s - r}{r}\nabla^2_\Omega\right)(\Psi + \Phi)dr\end{aligned} \tag{49}$$

- Density: δ_m
- RSD: $\frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$
- Lensing: $-\frac{1}{r_s}\int_0^{r_s}\frac{r_s-r}{r}\nabla^2_\Omega(\Psi+\Phi)dr = -2\kappa$
- Doppler: $\left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2}{r_s\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}$
- Potential: $\Psi - 2\Phi + \frac{\Phi'}{\mathcal{H}} + \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2}{r_s\mathcal{H}}\right)\left(\Psi + \int_0^{r_s}(\Psi' + \Phi')dr\right) + \frac{2}{r_s}\int_0^{r_s}(\Psi+\Phi)dr$

Galaxy Number Counts

Flux Limit

A telescope has a finite sensitivity and cannot see objects that emit light below a given flux limit $F_\star$ depending on the telescope.

- Apparent magnitude m

$$m_\star = -\frac{5}{2}\log_{10} F_\star + \text{const} \tag{50}$$

  where the constant is traditionally defined such that the star Vega has apparent magnitude zero.

- Luminosity L, is the total outward flow of energy from a radiating body per unit of time.
- Flux F, is defined as the total flow of light energy perpendicularly crossing a unit area per unit of time.

$$F = \frac{L}{4\pi d_L^2} \qquad d_L = (1+z)r \tag{51}$$

Define

- $n_s(z, \mathbf{n}, \ln L)$: The comoving number density at the source in a logarithmic interval of luminosity
- $\mathcal{N}_s$: The accumulative function of n_s

$$\mathcal{N}_s(z, \mathbf{n}, \ln L_\star) = \int_{\ln L_\star}^{\infty} n_s d\ln L \qquad (52)$$

- $N_s(z, \mathbf{n}, \ln L_\star)$: The (physical) number of sources per z per solid angle as measured by the observer [2]

$$N_s(z, \mathbf{n}, \ln L_\star) = \frac{r^2}{H}\mathcal{N}_s(z, \mathbf{n}, \ln L_\star) \qquad (53)$$

[2] Remember that $r = \int dz/H$

Given an observational threshold $F_\star$ in flux, we have

$$\mathcal{N}_s(z, \mathbf{n}, F_\star) = \int_{F_\star}^{\infty} \frac{\mathcal{N}_s}{dL} \frac{dL}{dF_o} dF_o \tag{54}$$

we found that

$$F_\star = \frac{L_\star}{4\pi D_L^2} = \frac{L_\star}{4\pi \mathcal{D}_L^2} \frac{\mathcal{D}_L^2}{D_L^2} = \frac{L_\star(1 + 2\delta_D)}{4\pi \mathcal{D}_L^2} \tag{55}$$

$$\mathcal{N}_s(z, \mathbf{n}, L_\star(1 + 2\delta_D)) = \mathcal{N}_s(z, \mathbf{n}, L_\star) - 5p\mathcal{N}_s(z, \mathbf{n}, L_\star)\delta_D \tag{56}$$

The fluctuation of luminosity distance is given by

$$\begin{aligned} \frac{\delta \mathcal{D}_L}{D_L} = &-\Psi - \left(1 - \frac{1}{\mathcal{H} r_s}\right) \left[\Psi + \mathbf{v} \cdot \mathbf{n} + \int_0^{r_s} (\Psi + \Phi)' dr\right] \\ &+ \frac{1}{r_s} \int_0^{r_s} \left(1 - \frac{r_s - r}{2r} \nabla_\Omega^2\right) (\Psi + \Phi) dr \end{aligned} \tag{57}$$

Magnification Bias:

$$s(z, m_\star) = -\frac{2}{5}\frac{\partial \ln \bar{\mathcal{N}}_s(z, L_\star)}{\partial \ln L}\bigg|_{L=L_\star} \tag{58}$$

Magnification bias quantifies the change in the observed number of galaxies gained or lost by lensing magnification

Since the process of galaxy formation is due to local physics, and since we expect our sources to follow the same velocity field as the dark matter, the clustering bias relation should be applied in the synchronous comoving gauge.

Clustering Bias [3] :

$$\delta_s = b(z)\delta_{mc} + \left(\frac{\partial \ln \bar{\mathcal{N}}_s}{\partial \eta} - 3\mathcal{H}\right) v_s = b\delta_{mc} + (b_e - 3\mathcal{H})\, v_s \qquad (59)$$

[3] $\delta_{mc} = \delta_m + 3\mathcal{H} v_s$, $\delta_{rc} = \delta_r + 4\mathcal{H} v_r$, where v_s is the velocity potential

Bias

Evolution Bias[4]

Evolution Bias:

$$b_e(z, m_\star) = \frac{\partial \ln \bar{\mathcal{N}}_s(z, L_\star)}{\partial \ln a} = -\frac{\partial \ln \bar{\mathcal{N}}_s(z, L_\star)}{\partial \ln(1+z)} \tag{60}$$

Evolution bias quantifies the physical change in the galaxy number density relative to the conserved case. Another point of view[1]:

$$\frac{N'}{N} = (1 - b_e/3)\frac{\rho'}{\rho} \tag{61}$$

Galaxy Number Density Flucuation

$$\Delta(\mathbf{n}, z) = b\delta_{mc} + (3 - b_e)\mathcal{H} v_s + \Psi - (2 - 5s)\Phi + \frac{1}{\mathcal{H}}[\Phi' + \partial_r(\mathbf{V} \cdot \mathbf{n})]$$
$$+ \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2 - 5s}{r_s \mathcal{H}} + 5s - b_e\right)\left(\Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{r_s} (\Psi' + \Phi') dr\right)$$
$$+ \frac{2 - 5s}{2r_s}\int_0^{r_s} \left(2 - \frac{r_s - r}{r}\nabla_\Omega^2\right)(\Psi + \Phi) dr \tag{62}$$

The kinematic dipole due to the velocity $\mathbf{v}_o$ in Δ_N is given by

$$d_n^{\mathrm{kin}} = \left(2 + \frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2 - 5s}{r_s \mathcal{H}} - b_e\right)\mathbf{v}_o \cdot \mathbf{n} \tag{63}$$

which we have negelcted before

Bias

$$\Delta_N = \Delta_n - b_e \Delta_e + 5s\Delta_s \tag{64}$$

where

$$\begin{aligned}\Delta_n = {} & b\delta_m + \Psi - 2\Phi + \frac{1}{\mathcal{H}}[\Phi' + \partial_r(\mathbf{V}\cdot\mathbf{n})] \\ & + \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2}{r_s\mathcal{H}}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_s}(\Psi' + \Phi')dr\right) \\ & + \frac{1}{r_s}\int_0^{r_s}\left(2 - \frac{r_s - r}{r}\nabla_\Omega^2\right)(\Psi + \Phi)dr\end{aligned} \tag{65}$$

$$\Delta_e = \mathcal{H}v_s + \left(\Psi - \mathbf{V}\cdot\mathbf{n} + \int_0^{r_s}(\Psi' + \Phi')dr\right) \tag{66}$$

$$\begin{aligned}\Delta_s = {} & \Phi + \left(1 - \frac{1}{r_s\mathcal{H}}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_s}(\Psi' + \Phi')dr\right) \\ & - \frac{1}{2r_s}\int_0^{r_s}\left(2 - \frac{r_s - r}{r}\nabla_\Omega^2\right)(\Psi + \Phi)dr\end{aligned} \tag{67}$$

Bias

$$\Delta_N = \Delta_n - b_e \Delta_e + 5s\Delta_s \tag{68}$$

$$\Delta_n = b\delta_m + \Psi - 2\Phi + RSD + \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2}{r_s\mathcal{H}}\right)\epsilon + 2\gamma + 2\kappa \tag{69}$$

$$\Delta_e = \mathcal{H}_s v_s - \epsilon \tag{70}$$

$$\Delta_s = \Phi + \left(1 - \frac{1}{r_s\mathcal{H}}\right)\epsilon - \gamma + 2\kappa \tag{71}$$

where

$$\epsilon = [\Psi - \mathbf{V}\cdot\mathbf{n}]_s^o + \int_0^{r_s} (\Psi' + \Phi')dr \tag{72}$$

$$\kappa = \int_0^{r_s} \frac{r_s - r}{2r_s r}\nabla_\Omega^2(\Psi + \Phi)dr \tag{73}$$

$$\gamma = \frac{1}{r_s}\int_0^{r_s} (\Psi + \Phi)dr \tag{74}$$

$$RSD = \frac{1}{\mathcal{H}}[\Phi' + \partial_r(\mathbf{V}\cdot\mathbf{n})] \tag{75}$$

Calculation

Transfer Function

Expand curvature perturbation and number count fluctuations

$$\Phi_{\mathsf{MD}} = \sum_{l=1}^{\infty} \frac{1}{l!} \Phi_{\mathsf{MD},l} (kr\cos\theta)^l \tag{76}$$

$$\Delta_N^X = \sum_{l=1}^{\infty} \frac{1}{l!} D_l^X (\cos\theta)^l \tag{77}$$

Transfer functions

$$\Phi(a) = T_\Phi(a)\Phi_{\mathsf{MD}} \tag{78}$$

$$v(a) = T_v(a)\Phi_{\mathsf{MD}} \tag{79}$$

$$T_\Phi(a) = \frac{5\Omega_m}{2} \frac{\mathcal{H}}{\mathcal{H}_0 a^2} \int_0^a \frac{da_1}{(\mathcal{H}(a_1)/\mathcal{H}_0)^3} \tag{80}$$

$$T_v(a) = \frac{5\Omega_m}{2} \frac{1}{\mathcal{H}_0 a^2} \int_0^a \frac{a-a_1}{a_1} \frac{da_1}{(\mathcal{H}(a_1)/\mathcal{H}_0)^3} \tag{81}$$

Calculation

M.D. universe case

Shear-free: $\Phi \approx \Psi$

M.D. universe: $\Omega_m = 1, \mathcal{H} = 2/\eta$

Transfer function:

$$T_\Phi(a) = 1 \qquad T_v(a) = \frac{\eta}{3} \tag{82}$$

Contribution of $l = 1$ term:

$$\Delta^e = 0$$
$$\Delta^s = 0$$
$$\Delta^n = 0$$

Calculation

M.D. universe case

Contribution of $l = 2$ term:

$$\Delta^e = \left(\frac{1}{6} - \frac{1}{3}\frac{\eta_o}{r}\right) k^2 r^2 \cos^2\theta \Phi_{\mathsf{MD,2}}$$

$$\Delta^s = -\frac{1}{6}k^2 r^2 + \left(-\frac{1}{12}k^2 r^2 + \frac{5}{12}k^2 r \eta_o - \frac{k^2\eta_o^2}{6}\right)\cos^2\theta$$

$$\Delta^n = -\frac{1}{3}k^2 r^2 + \left(-\frac{1}{12}k^2 r^2 + \frac{2}{3}k^2 r \eta_o - \frac{k^2\eta_o^2}{2}\right)\cos^2\theta$$

Contribution of $l = 3$ term:

$$\Delta^e = \left(\frac{k^3 r^3}{9} - \frac{1}{6}k^3 r^2 \eta_o\right)\cos^3\theta$$

$$\Delta^s = -\frac{1}{12}k^3 r^3 \cos\theta + \left(-\frac{1}{12}k^3 r^3 + \frac{1}{4}k^3 r^2 \eta_o - \frac{1}{12}k^3 r \eta_o^2\right)\cos^3\theta$$

$$\Delta^n = -\frac{1}{6}k^3 r^3 \cos\theta + \left(-\frac{1}{6}k^3 r^3 + \frac{7}{12}k^3 r^2 \eta_o - \frac{1}{3}k^3 r \eta_o^2\right)\cos^3\theta$$

Calculation

M.D. universe case

$$D_1^e = 0 \qquad D_1^s = -\frac{1}{12}k^3r^3\Phi_{\mathsf{MD},3} \qquad D_1^n = -\frac{1}{6}k^3r^3\Phi_{\mathsf{MD},3}$$

$$D_2^e = \left(\frac{1}{3} - \frac{2}{3}\frac{\eta_o}{r}\right)k^2r^2\Phi_{\mathsf{MD},2}$$

$$D_2^s = -\left(\frac{1}{6} - \frac{5}{6}\frac{\eta_o}{r} + \frac{1}{3}\frac{\eta_o^2}{r^2}\right)k^2r^2\Phi_{\mathsf{MD},2}$$

$$D_2^n = -\left(\frac{1}{6} - \frac{4}{3}\frac{\eta_o}{r} + \frac{\eta_o^2}{r^2}\right)k^2r^2\Phi_{\mathsf{MD},2}$$

$$D_3^e = \left(\frac{2}{3} - \frac{\eta_o}{r}\right)k^3r^3\Phi_{\mathsf{MD},3}$$

$$D_3^s = -\left(\frac{1}{2} - \frac{3}{2}\frac{\eta_o}{r} + \frac{1}{2}\frac{\eta_o^2}{r^2}\right)k^3r^3\Phi_{\mathsf{MD},3}$$

$$D_3^n = -\left(1 - \frac{7}{2}\frac{\eta_o}{r} + 2\frac{\eta_o^2}{r^2}\right)k^3r^3\Phi_{\mathsf{MD},3}$$

References

- Ruth Durrer.
  The Cosmic Microwave Background.
  Cambridge University Press, 2 edition, 2020.
- Antony Lewis and Anthony Challinor.
  Weak Gravitational Lensing of the CMB.
  Physics Reports, 429(1):1–65, June 2006.
  arXiv:astro-ph/0601594.
- Camille Bonvin and Ruth Durrer.
  What galaxy surveys really measure.
  Physical Review D, 84(6):063505, September 2011.
  arXiv:1105.5280 [astro-ph].
- Anthony Challinor and Antony Lewis.
  The linear power spectrum of observed source number counts.
  Physical Review D, 84(4):043516, August 2011.
  arXiv:1105.5292 [astro-ph].